Running Head: COMPLEXITY MODEL OF ENTREPRENEURSHIP

Generative Conditions for Entrepreneurship:

A Complexity Science Model

By

James K. Hazy

Jeffery Goldstein

hazy@adelphi.edu

goldstei@adelphi.edu

Abstract

Complexity science is used to describe innovation and entrepreneurship. *Context* is addressed by positioning entrepreneurship at the nexus of two categories of constraints: The rate that resources can be appropriated creates tension to organize to exploit opportunities, and the rate that information becomes available challenges organizing efforts in an evolving, specialized, and distributed environment. The complexity phenomenon of *emergence* is the key enabler of entrepreneurship, while recombination of capabilities, technologies and social networks is a mechanism used by individuals to construct a better way to exploit the opportunity. We deduce testable propositions and suggest further research.

Generative Conditions for Entrepreneurship: A Complexity Science Model

1. Introduction

Entrepreneurship has become increasingly important globally mostly due to three factors: its indispensible role in enabling innovation; its capacity for creating new jobs and accumulating capital; and its function as a powerful engine for social change directed at pressing issues in healthcare, climate change, poverty, and so on (Dacin, Dacin, and Matear, 2010; Goldstein, Hazy, and Silberstang, 2010). Each of these factors has not always received the independent attention that it deserves due to the way that entrepreneurship is traditionally studied, most notably as a business process for creating wealth in a purely economic context. For example, entrepreneurship is often studied in terms of the business opportunities that are pursued - how such opportunities are recognized and exploited (Eckhardt and Shane, 2003; Sarason et al., 2006), why certain ones are chosen and others are not (Lee and Venkataraman, 2006), or whether opportunities are recognized or constructed (Vaghely and Julien, 2010). Considerably less is known about how exogenous contexts generate conditions by which entrepreneurial ventures become the site of innovative organizing processes.

To address such a knowledge gap, this paper offers a complexity science-based mathematical model to understand the interrelationship between rapid changes in environmental contexts and concomitant shifts in internal modes of organizing capable of quick adaptation to these rapidly changing contexts. In our model this enhanced capacity for adaptability hinges on two parameters: better resource exploitation through innovative modes of organizing; and better access to the kind of information that allows for enhanced resource exploitation.

Complexity science has emerged over the past several decades as a set of powerful methods and constructs yielding greater understanding of a diverse set of complex and highly interdependent phenomena ranging from technological innovation (Arthur, 2009; Fleming and Sorenson, 2001), to animal and insect colonies (Camazine et al., 2001), to lasers and other coherent physical processes

(Haken, 2006; Laughlin, 2006), to chemical reactions (Nicolis and Prigogine, 1989), to economies (Krugman, 1996), to organizational change (Goldstein, 1988, 1990), and to entrepreneurship, both in a general way by McKelvey (2004), and in a specific way involving the complexity notion of emergence (Lichtenstein et al., 2006, 2007). We are proposing here that the methods, constructs, and insights of complexity science can shed light on the social, technological and organizational innovations associated with entrepreneurship, particularly the diverse mechanisms of novelty generation which are at the heart of successful entrepreneurial ventures.

We argue that generative conditions for entrepreneurial innovation arise due to the interaction of two distinct constraints or contexts that are parameterized in our model. The first involves the potential for accelerated access to resources including additional sales or margin, i.e., the *rate of resource availability (RRA)*. The second involves the accessibility of information relevant for taking advantage of an opportunity or addressing a problem, i.e., the *rate of accessible information (RAI)*. Our model describes how generative conditions develop as these parameters interact to create well-studied complexity conditions called bifurcation and *criticalization* which are necessary for system-wide innovation and change. Criticalization, in turn, opens the way for the experimentation and novelty generation associated with the phenomenon of emergence in complex systems. It is through criticalization that entrepreneurial ventures are able to challenge the current status quo or "stable equilibrium" via innovation.

Although our model is conceptual in nature, it represents a critical step toward a more complete systemic rendering of those organizing dynamics involved in entrepreneurial innovation. Accordingly, we suggest seven propositions for subsequent empirical exploration.

2. The constructs and methods of the model

Because most research into organizations in general has been limited to stable systems that can be approximated using linearization techniques[1], deep seated innovation has often defied analysis since it does not normally arise under conditions of stability where linear thinking works. The methods and constructs of complexity science, however, focus on nonlinear systems where linear approximation breaks down in the face of instability. Moreover, by appealing to the mathematical underpinnings of complexity science, our model can play a role in the reinvigoration of the use of mathematical models for researching entrepreneurism as advocated recently by Lévesque (2004). Indeed, our model offers a direct mathematical representation of the dynamics of entrepreneurial innovation and is not limited to the use of statistical analyses supporting what are essentially non-mathematical arguments.

It is helpful to distinguish our mathematical approach from the equilibrium-based framework typically used in economic modeling (Henderson and Quandt, 1980). McKelvey (2004) has pointed out that equilibrium assumptions are prevalent even in evolution-theoretical approaches to entrepreneurship. Our model, though, follows complexity science by including provisos for dealing not just with equilibrium conditions that tend toward restored stability after perturbations but also with non-equilibrium conditions that tend toward increasing instability such as market changes or technology disruptions. The latter non-equilibrium conditions can lead to qualitative change and transformation since it is the latter and not the former within which innovation becomes a possibility.

Second, our approach differs from economic modeling in the manner in which contextual shifts in the environment are modeled. Traditional methods treat these as "exogenous shocks" to the economy which are modeled as coefficients on error-correcting supplementary equations such as changes in the purchase price of investment goods or as an adjustment cost function (Devarajan and

---

[1] The interested reader should explore the precise conditions where this very important and ubiquitous assumption actually holds in dynamical systems. This is important because the requisite conditions impact on the validity of mathematical analyses that are used every day in organizational settings. The details of linearization are defined in the Linearization Theorem which is found in Hirsch, Smale and Devaney (2004: 168) among other texts.

Delfin, 1998; Rumler, 1999). In equilibrium-based economics the internal or endogenous dynamics of the economic system are modeled as a steady state wherein supply and demand values are said to possess "total endogeneity" since their values are supposed to play off of each other in such a way that the system will eventually return to equilibrium even after undergoing exogenous shocks. In contrast, we parameterize external effects directly as coefficients on the dynamical equations in our model.

As we will show, such a steady state model is clearly not adequate to capture the instability of criticalization that is necessary for the adaptive innovation that characterizes entrepreneurial firms. In particular, our model employs three interrelated complexity science constructs:

1. *Nonlinear dynamical systems theory* ("nds"): dynamical equations (differential and difference) used to describe the transition of an entrepreneurial venture's internal state through different *attractor regimes*, each reflecting a distinct, stable organizing mode; (see Hirsch, et al, 2004; Scott, 2005; Tufillaro, et al, 1992).

2. *Bifurcation theory including catastrophe theory* (a subset of bifurcation theory) that expresses the various ways that systems transition into new attractor regimes, a situation usually termed *criticalization* (Arnold, 1991; Thom, 1989; Zeeman, 1977). It is important to note that in the 1970s when Zeeman popularized the theorems of Thom and others, controversy over the appropriateness of certain applications soured the reputation of catastrophe theory for a time. Since then, however, the legitimacy of catastrophe theory has been reestablished (Arnold, 1991; Guastello, 1992).

3. *Emergence* in complex systems which involves the coming into being of new structures that are stable and sustainable with novel properties, usually understood as taking place at bifurcations (see Goldstein, 1999; Holland, 1998); emergence is characterized in large measure by *recombinatory* operations which have been implicated in many studies of innovation and invention (Arthur 2009).

A complexity science approach using these elements also addresses the call for a *systemic* perspective to entrepreneurship along the lines sketched-out by Morlacchi (2007). It not only

recognizes the crucial role that is played by the many differences at work in the formation of entrepreneurial ventures - different players in different interactions within different institutional contexts operating according to different norms, different regulations, and so on - but also the need to integrate this diversity in theory to provide practical guidance to entrepreneurs and policy makers.

Our approach follows the modeling scenarios advocated by pioneers in complexity science, e.g., the Nobel Laureate Ilya Prigogine who put forward the following tenet over twenty years ago: "The evolution of such a system [human systems] is an interplay between the behavior of its actors and impinging constraints from the environment" (Nicolis and Prigogine, 1989: 238). Our model assumes that context matters substantively and that innovative solutions to developing opportunities are unlikely to emerge out of a market economy absent very specific constraining conditions. Indeed, the mathematics of bifurcation demonstrate that absent constraints, systems can be reoriented so that crossing bifurcation points where radical change becomes possible are so improbable as to be irrelevant (Arnold, 1991; Thom, 1989). This also makes intuitive sense. If the system is not constrained at all, the requisite conditions for qualitative change never happen because the individuals in the system, being unconstrained, just maneuver around difficulties. Constraints -- like the need for seed capital, or capacity restrictions in a factory -- limit this ability of individuals to maneuver within the system to the point where qualitative change becomes inevitable.

Another important finding from complexity science backs this up, namely, that the external containers in which "self-organizing" processes occur play a critical role in the patterns that emerge (see Berge et al., 1984; and Weiss, 1987). But there are also differences between self-organizing in physical systems and innovative reorganizing in human interaction dynamics. In the latter case which includes entrepreneurship, there is an added benefit for researchers since the dynamics of innovation can be observed directly. Unlike physical systems, innovative organizing approaches are intentionally constructed by the parties involved. Thus, while our model is informed by the study of self-organizing

process models from the physical sciences, our findings in the human system context may also eventually inform fundamental research in the natural sciences.

2.1 A nonlinear dynamical systems framework for innovation

Nonlinear dynamical systems theory in mathematics (nds) has proven to be an effective approach in previous studies of the evolution of technology innovations (e.g., See Arthur, 2009; Arthur and Polack, 2004; Fleming, 2001; Fleming and Sorenson, 2001; Smith, 2002). Moreover, there are many precedents for analyzing outcomes to social events using nds methods, e.g., seminal studies in epidemiology and political conflict (Epstein, 1997) and the analyses of social contagion across social networks (Dodds and Watts, 2005). A simple and important nds example is provided in Appendix A.

An nds model is framed according to both *internal variables* of interest - in epidemiology, for instance, infected versus susceptible populations-- and *external parameters* representing external constraints - like population growth rates or how easily an infection is transferred in an interaction. In the case of our model, these external parameters represent contextual constraints affecting the success potential of entrepreneurial programs. We make the argument that under certain parametric cases, stability dominates and therefore entrepreneurship is unlikely or impossible. In other cases the presence of entrepreneurship is likely although the specificity of outcomes always remains less certain.

A dynamical system model describes how the range of values for the internal variables changes with respect to one another over time as the parameter values change. It is crucial to recognize the difference between variables and parameters in this kind of modeling strategy because we posit that it is the parameters, and not the variables, that provide the generative context which drives the social, economic and technical innovations that are required by entrepreneurship. Internal variables like worker productivity, sales growth, or cash flow are not sufficient drivers for innovation since it is the parameters that determine the possible values for these variables and not vice versa. The reason for this has to do with the relation between parameters, attractors, and the emergence of new attractors.

Values of the internal variables and their rates of change are represented in a *state* or *phase space,* stable subsets of which are called the *attractors* (or attracting sets or attractor regimes) of the system. Once the system enters the "basin" of an attractor the variables are determined by ranges of values permitted by the attractor set. For example, if the system's parameters are constant, a system or organization can be caught within a particular attractor regime, so that the system is resistant to change meaning that after the perturbations subside the system will relax back into its stable attractor. A stylized illustration of a stable attractor in a three dimensional projection of phase space that reflects three system variables is shown in Figure 1. Variables might include sales growth, operating margin and employee productivity and their rates of change. The idea is that these variables and their rates of change are interrelated, codetermining one another's range of values which then have to stay within the range determined by the attractor regime.

|Insert Figure 1 about here |

The nature of attractors are investigated in mathematics according to what has been termed their "qualitative dynamics" or "qualitative analysis", i.e., the qualitative distinction between specific geometries and topologies of the attractive sets in phase space (Tufillaro et al., 1992). The "jump" in qualitative dynamics from one attractor to another is used in our model to refer to the "jumps" in qualitative novelty characterizing different modes of organizing found in entrepreneurial ventures. Note that the qualitative jumps are not explained in traditional economic models and that these jumps are intuitively similar to leaps of innovation and entrepreneurship.

2.2 Bifurcation and Criticalization Makes Innovation Possible

One of the most far-reaching findings in nds is that changing the values of the parameters (constraints, contexts, containment) can in some cases change the attractor regime governing the system by "forcing" a bifurcation of the system into new attractor regimes. This results in a changing pattern of variables. At bifurcation, what was stable becomes unstable. Figure 2 illustrates how a stable

regime (outside the cusp-shaped area) bifurcates into two (in the cusp) when one of the two parameters, *b*, changes along the abscissa and the other, *a*, changes along the ordinate.

At the bifurcation point for *b*, the system becomes unstable, literally "forking" into an unstable region of parametric ordered pains (*a, b*) to the right. These are the parametric values that imply criticalization and thus offer the potential for innovation (Goldstein et al., 2010). When both parameters *a* and *b* are such that the system is within the cusp, two possible attractors or two stable organizing approaches for the internal variables - for example a fast growth path and a slow or flat growth path - may coexist within the same venture:

| Insert Figure 2 about here |

It is important to notice that the parameter *b* on the abscissa is not representing time as it often does on the horizontal axis of a time series. The passage of time by itself need not be a driver of bifurcation.

When the parametric conditions signal a transition to a new attractor, the system is said to be in *criticalization*, a state exhibiting two or more distinct attractors or regimes of stability, e.g., one might represent a continuation or incremental modification of prior routines, such as the gradual growth of existing markets, while the other might be entrepreneurial innovation leading to new organizing activities, new strategies, or new products/services, e.g., entrance into or construction of new markets. Criticalization can be recognized phenomenologically by a sense within the system that prior ways of doing things no longer apply, a state of confusion stemming from the fact that two or more distinct approaches are being operationalized at once (Goldstein et al., 2010). As we describe in Section 3, we are understanding bifurcation, and thus criticalization, in terms of the mathematical construct of catastrophes, i.e., "cusps" of change, which have the advantage of demonstrating simultaneously both an incremental and a sudden onset of innovation depending upon parameter values.

To summarize, our model hinges on three fundamental constructs: i) a parameterization of dynamical systems' equations that represent two main aspects of context (to be explicated in Section

2.1.1) that affect entrepreneurial ventures; ii) the phenomenon of criticalization associated with bifurcation in dynamical systems when the parameters, in our case, the two parameters, exceed identifiable threshold values; and iii) a catastrophe theory perspective on criticalization that allows both for incremental and precipitous change. This model translates into a theory of how changes in environmental constraints or contexts enable a shift from the stability of an original attractor regime representing existing routines, practices, capabilities, social networks, and so on, of "business as usual" to the emergence of new attractor regimes representing substantive transformations of routines, practices, capability, social networks, and so on, i.e., "business as innovative".

2.2.1. The two parameters that drive innovation

In our model, institutional and community constraints or contexts act on the social system to drive entrepreneurial innovation. More specifically, we posit that there are two critical parameters pertinent to entrepreneurial innovation; the *rate of resource appropriation (RAA),* what we call parameter *b*, and the *rate of accessible information (RAI)* which we call parameter *a*.

The first parameter *RRA,* the rate of resource availability, is a metric related to what has previously been described as "adaptive tension" (Uhl-Bien et al., 2007) or "opportunity tension" (Goldstein et al., 2010). The values taken by *RRA* reflect the recognition that when the environment presents improved access to necessary resources including better sales or profits, these resources only become fully exploitable to the extent that people in the system organize to acquire them. In effect, the potential to acquire additional resources generates a need for coordination that was not previously forthcoming. Stated another way, resources can only be appropriated in a manner that has as much differentiation, correlation, and variety as the environment itself possesses, an analogue to Ashby's (1956) insight that adaptable systems must have a variety requisite to their environments.

RRA is described as an adaptive or opportunity *tension* since it presumes a context in which a problem or opportunity implies the availability of additional resources. However, there is a gap

between actual and potential performance because the availability of resources also depends upon the generation of innovative modes in which people and technology are organized. To illustrate, when agents act *alone*, the environment might have a particular level of resource availability. At the same time, however, there might be another, greater level of resource availability that is only exploitable if individuals act in *collective unison*, and this requires organizing.

For example, with individuals hunting alone, only small game may be available for hunting. Yet, additional prey - a potential for significantly more resources - in the form of larger game like a fast running stag might become available to the group, but these resources remain out of reach to individuals acting alone. The latter resource availability potential creates an opportunity, but only if a hunting party can be formed, that is, only if an innovative collective activity can be organized. A changing environmental context and a new mode of organizing work together to imply an increase in the value of *RRA,* leading to a proposition:

Proposition 1: As *RRA* increases beyond a certain threshold so that criticalization at bifurcation ensues, this will be accompanied by a new set of perceptions among the venture's players involving the benefits of organizing differently, that is, acting in greater collective unison rather than acting alone.

Once an appropriate set of metrics is established for measuring RRA, this proposition could be operationalized through the use of surveys, interviews, narrative analysis, and other methods of assessing perceptual and attitudinal change.

The second parameter, *RAI,* measures the capacity to exploit resources more effectively by an increase in the accessibility of pertinent information. *RAI* refers to what could be called informational *differences* following Bateson's (2000) well-known definition of information as *a difference that make a difference*, in other words, that diversity in perspectives and knowledge which Page (2007) has proven, by way of computational simulations and theorems, fuels creativity. In terms of innovation, informational differences refer to the multifarious differences in outlooks, know-how, connectivities,

and so on that are present among the social actors facing the market problem or opportunity but which may not be fully accessible for a host of reasons. When conditions are changing rapidly, improvements in access to this information as well as an increase in the capacity for gathering, processing and using information (Gell-Mann, 2002) can become an indispensable prod for venture formation or innovative new directions.

RAI is a crucial parameter. Even in the case when there is a high level of *RRA,* entrepreneurial innovation may not ensue if the relevant informational differences to address the problem are too widely dispersed, or if they are not accessible to be parsed and processed in a cogent fashion. Only those groups with a capacity to gather and use the relevant information about opportunities can organize to exploit them (Kirzner, 1985). If the social system is to use these informational differences to innovate in accord with the new attractor possibilities, its members must be in a position to sense, recognize, gather, amplify, interpret, disseminate and then synthesize the information effectively *as a system*. For instance, pertinent and useful information about how to best approach a market problem may be held by individuals who may not be in a position to influence decisions about organizing a response to the opportunity. Thus, as we discuss in Section 4.1, *RAI* is intimately connected to the nature and quality of social networks and their effectiveness at transmitting informational differences.

It is also important to recognize what the term "differences" implies in the context of *RAI.* In information theory, Shannon had originally formulated his metric on the amount of information in a communication channel in terms that emphasized the surprise or *unexpectedness* of a message, unexpected in the sense of a departure or difference from expected redundancies in the observations (see Mackay, 1969). Thus, messages composed entirely of redundancies would have little information content. There would be nothing new. Later, as we alluded to earlier in this section, Bateson (2000) and Page (2007) expanded on the notion of difference as a specific quality of information.

Understanding informational differences and thereby *RAI*, in this way might, however, at least

on first impression, be seen as somehow contrary to the coordinative/cooperative implications of *RRA* since cooperation would appear to connote the opposite of differences or divergence. Seeing why this opposition is only apparent and not substantive can be helpful in better appreciating just what *RRA,* as involving innovative organizing for the purpose of greater exploitation of resources, entails. For although we have had occasion to describe examples of *RRA* as collective unison, such coordination and cooperative connotations of RRA are just one particular way that *RRA* might manifest itself. More to the point, we intend *RRA* to indicate any kind of innovative change in the modes used to organize people, knowledge and technology to make resources more available for appropriation. This might include cooperation, but it would be a kind of cooperation that maintains the differences in skills and perspectives among the cooperating agents that are critical when maintaining the potential for generating creative novelty. The innovation associated with *RRA* might also show-up in a multitude of other types of organization, for example, modularity in the structure of organizing activities, or even some forms of hierarchy and specialization (Hazy, 2008) or even arms-length market transactions (Williamson, 1975) that more effectively render environmental resources available or accessible.

Furthermore, by understanding that differences are key to both *RAI* and *RRA*, it is easier to understand how at criticalization, both of these parameters are closely associated with the phenomenon of emergence which hinges on the novelty generating potential of differences in a complex system. As we show in Section 4, with emergence these recombine to create entirely new modes of organizing.

The *RAI* parameter is so crucial to entrepreneurial innovation that even if *RRA* has pushed the system toward criticalization, bifurcation may not take place if *RAI* does not itself cross its own critical threshold. That is, a "jump" in innovation is only possible when the *RAI* threshold is crossed since an alternate, stable organizing model of operations can only come about as informational differences are recognized and disseminated. These remarks lead us to a further proposition:

Proposition 2: When the parameter *RRA* has increased in value even to the point where it might have

been thought that criticalization would ensue, this would not happen unless *RAI* has also increased beyond a critical threshold.

The same operationalizing issues pertain here as stated above in relation to Proposition 1.

3. Innovation in the cusp catastrophe mode

To understand why a new stability regime can emerge during criticalization, it is useful to look at results from another but closely related area of mathematics that hones in on bifurcation dynamics, namely, the field of catastrophe theory (Arnold, 1991; Thom, 1989 Zeeman, 1977). First, though, let's consider how traditional linear approximation schemes might attempt to predict the future state of an organization (Hirsch, Smale, and Devaney, 2004). During times of *stability*, it could be thought that additional factory hours should lead to proportionally more output, which in turn should lead to proportionally more sales, and so forth; or additional investment in innovative marketing campaigns may lead to incremental sales increases, or investment in product development can lead to new features that extend or improve product performance proportionally. The enmeshment of stability with linear analysis also implies that a particular attractor regime has a staying potency due to its ability to dampen deviations, e.g., a drop in profitability could be met with some sort of remediation like a cost reduction program, or a decline in sales may prompt the hiring of a new sales manager, and so forth.

When a social system undergoes criticalization, however, there is a growing recognition that operating models that worked before are not sufficient in either resolving pressing problems or in taking advantage of newly presenting opportunities. From an nds perspective, this means that the reigning attractor and its associated mode of organizing are no longer adequate, and that what is needed is an innovative "jump", a situation that can be modelled using the construct of a *cusp catastrophe* shown in Figure 3 (Zeeman, 1977). It is important to note three things about Figure 3; first, it is not a phase space for depicting attractors as in Figure 1 but a different kind of "space" that shows bifurcation in terms of changes in two parameter values; second, this model is only appropriate

as a representation of *local* dynamics near the point of bifurcation (Arnold, 1991; Thom, 1989) and thus does not imply what may be happening elsewhere in the system away from bifurcation values of the parameters; third, it displays the cusp-shaped bifurcation curve at the bottom that results from a two dimensional projection of the three dimensional surface above it as if a light was shining from above downwards on the cusp and the bifurcation curve at the bottom was a shadow (this is done to show the connection between the cusp catastrophe and the bifurcation shown in Figure 2.)

We are turning to the construct of the cusp catastrophe in particular in order to model the kind of innovation generated by our model's two parameters for two reasons. First, Thom (1989) has described how it was mathematically proven that the cusp catastrophe is the unique model that describes the simplest case of nonlinear change from one stable topological form to another, the simplest case being one with a single internal variable and two parameters. To be sure, organizations are much more complex than would seem to be reflected when assuming a single variable is of interest, but this is just a start, and as we describe in Appendix A there is reason to be confident that the approach will provide useful insights[2]. Second, the cusp catastrophe construct has intuitive validity in that it can exhibit both incremental and sudden change through innovation, a feature that we will exploit in Section 5 when discussing Figure 6.

The cusp catastrophe represents those parametric conditions resulting in a bifurcation or splitting of the pre-existing "business as usual" state into two or more new states, each representing a novel approach to organizing that can act as an attractor for the system. Business-as-usual becomes unstable represented by the dotted line in Figure 2. However, only one of these emerging stable states might make best use of the new potential to appropriate resources at an increased rate, and the potential can only be realized by implementing innovative ways to organize activities, resources and people.

---

[2] Significantly, higher degree mathematical models (which would be the subject of future research) would not be alternatives, but would rather extend this approach and offer more precise expressions of topological transitions with more finely differentiated parameters and variables. This can be deduced since work in this field has proven theorems that demonstrate that there are unique (but more mathematically involved) models for topological transitions for each case with up to two internal variables and four external parameters (Arnold, 1991; Thom, 1989).

The other attractor state represents an alternative approach - perhaps factions competing for resources - that may be less promising and fail to realize the full potential of the opportunity.

Bifurcation in the cusp of change can be described as follows: As the *RRA* parameter increases past a threshold, two possibilities open-up. Organizing activities form into a more coherent whole in order to capitalize on the emerging greater resources, or the collective fragments into stable collections of feuding groups. When the system becomes unstable, innovative change is possible since any small deviation from business-as-usual starts a process of transition to one of the other stable state, a process that relates to the seeds of novelty that can be amplified as we will discuss in Section 4. Which of these two end states occurs depends upon both initial conditions, e.g., the characteristics of the venture team, and the two parameters *RRA* and *RAI,* i.e., how well the team becomes coordinated and how well it senses and synthesizes information across the environment in order to organize more effectively.

The surface in Figure 3 of the cusp catastrophe illustrates the organizing solutions that have stable performance for each parametric setting. The specific solution depends upon the particular external conditions that are parameterized by *a* representing *RAI* and *b* representing *RAA*. The unshaded surface in the figure represents the subset of stable organizing states that, if achieved, would realize a stable process for appropriating resources. In fact, a mathematical solution exists for any ordered pair of the parameters *a* and *b,* (for any rate of resource appropriation and level of accessible information) and most often there is a single point. This may simply mean that everyone plays his or her part in collective activity. For the most part, the locally stable solution is also the best solution in terms of realizing the opportunity's continuing performance potential. Most often, routines work.

However, there are values inside the fold - this can be seen as in the cusp-shaped region on the 2-dimensional parametric plane (*a* is plotted against *b*) that was shown in Figure 1 - where there are two locally stable solutions, the solutions on the bottom of the fold and those on the top. Note that in these cases, even though both are stable the relative performance *Y* of the two solutions implies that the

solution on top of the fold exploits more of the potential than does the solution on the bottom. In this case, organizations starting on the bottom of the catastrophe diagram are where radical innovation could bring about qualitative jumps in organizing by "leaping" from the bottom surface up to the top[3]. As we describe in Section 5 and as Figure 6 shows, there is another path to innovation characterized by incremental change which is labelled in Figure 6 as pathway 2 which goes around the cusp by changing parameters to, in effect, go around the cusp.

Although we will say more about these two pathways to innovation later when we focus on Figure 6, it can be helpful now for the understanding the advantages offered by appealing to the construct of the cusp catastrophe to consider the example of a start-up SmartPhone application development company that has outsourced all of its functions. This company may become successful by finding a configuration of internal variables that is stable in the attractor as represented by the points on the bottom surface of the fold, and is incrementally optimized by continuous improvement. However, once the company relocates near a university, additional resource potential becomes available in the form of trained and available software developers. As the human capital resource constraint is relaxed, *RRA* increases beyond a bifurcation point such that another, and potentially better configuration of internal variables, becomes possible to exploit the same SmartPhone opportunity. Moreover, by bringing some or even all of the technical functions in house, the *RAI* parameter can actually be observed to increase since customer information can be more quickly processed and integrated into the applications. Thus, jumps in innovation are possible, but the venture may take small steps or larger ones to get there; both approaches are apparent in the model.

| Insert Figure 3 about here |

---

[3] This may seem reminiscent of the performance landscapes used in business strategy by Levinthal (1997), Siggelkow and Rivkin (2005) and others. However, although it is conceptually similar, we point out that in our model measureable external parameters are interacting to create the surface rather than simulated internal variables like genetic material or strategic choices. Further, this model is derived from the well-developed mathematics of bifurcation and catastrophe theory (Arnold, 1991) rather than being a graphical illustration of computational modeling simulations.

It is important to note that for a given metric for performance (shown in the figure as *Y*), there is most often (but not always) a best performance level for each parametric situation. The values of the internal variables that describe the various properties within a system may change (e.g., the "small fixes" mentioned above or employee turnover) but when performance is being maximized locally, the attractor in effect does not change as long as the external constraints stay the same. Thus for many parametric settings, routine work gets the job done. During these times, linear analysis is predictive. However, criticalization and instability are necessary if innovation must occur to take advantage of the opportunity or to resolve the pressing problem. To model innovation, a nonlinear approach is needed.

3.1 The generative interaction of the two parameters enables venture formation

To understand how the two parameters can interact, assume first a condition of adequate *RRA* such that a venture forming opportunity even exists, e.g., the opportunity to develop smart phone applications, for example. Then, regarding the *RAI* parameter, there are two extremes to consider. In one extreme, individuals are isolated with few network connections, any access to the developers' toolkit is problematic, individuals are unaware of how one might even acquire or use it, and it is unclear how one might gain insight or access to the market anyway. In this case, the presence of opportunity tension even beyond the threshold point of *RRA,* e.g., the opportunity for a software developer to build a new iPhone application, becomes irrelevant.

At the other extreme, connections are pervasive, rich and nuanced. In effect, everyone knows about the opportunity and has all of the information needed to build and market the applications, e.g., knowledge is present in incumbent firms as well as start-ups. In such a situation, established application development groups would organize quickly and effectively to close any opportunity tension gap. At this end of the scale, opportunities favor incumbent organizations because they possess complementary assets and can quickly take advantage of economies of scale and scope. This extreme is analogous to efficient markets in finance (Henderson and Quandt, 1980) when arbitrage

opportunities are eliminated almost immediately. To reach this condition, though, a second threshold on *RAI* must be passed to take the system out of the cusp of change such that informational differences are incorporated into activities so rapidly that choosing not to reorganize is no longer even an option.

Real world markets and organizations, of course, exist somewhere between these extremes. In most cases, information about resource availability has value, at least for a time. This creates opportunities within the cusp of change wherein the potential paths of action are determined by the interaction of the two parameters. First, *RRA* must be significant enough to justify the effort needed to galvanize social actors in the new ways needed to address the opportunity. Second, there must be necessary levels of *RAI*, that is, enough accessible informational differences concerning relevant knowledge about the opportunity. This allows for the emergence of a deeply nuanced operating model aimed at the opportunity by means of iterated interactions across the organization. (We describe this process in Section 4). The condition caused by sufficient levels of *RRA* and *RAI* form the crucible wherein the emergence of entrepreneurial social innovation occurs (Hazy, 2009). The cusp catastrophe and the bifurcation implicit in it suggest a proposition concerning the identification of the shift from one attractor (representing business as usual) to the new emerging attractors (representing innovation):

Proposition 3: Choosing some suitable internal variable that can be measured before and after some bifurcation event signifying the emergence of new attractors, there will be an identifiable difference in the qualitative dynamics among the attractor portraits in phase space.

The empirical test of this proposition would need to include two methodological presuppositions. The first involves the Takens embedding theorem mentioned in Appendix A which has proven that much of the interesting phase space dynamics of a nonlinear dynamical system can be captured by plotting only one variable of interest against delayed time values of itself. For instance, one such variable involving entrepreneurial innovation could be a count of instances of particular themes within business narratives, e.g., a theme about opportunity recognition was identified by

Lichtenstein et al. (2006) using textual analysis software. The second involves developing a method for determining what is called attractor *dimensionality* (e.g., fractal dimensionality, correlation dimension, box counting dimension, topological entropy, and others) for entrepreneurial ventures. As a starting point, over the past 20 years, several useful metrics regarding mathematical attractors have been developed (Jost, 2005). We posit that if these metrics can be applied to entrepreneurship, they could be used to identify significant differences in the attractors that emerge at criticalization as compared to those at work before criticalization.

3.2 Potential functions: Entrepreneurial innovation as a wave of perceived possibilities

Another way to understand bifurcation into new attractors is through what are called potential functions, the slope of which represents the direction and tendency for the system to change its functioning. A slope of zero therefore indicates points of stability. These points of stability can be associated with the attractors for the related nds that describes the organizing system. (For graphs of potential functions with different values of the parameter *a* with a constant value for *b*, see Figure 4.) We are interested in how the state of the system and its stability are affected as the system's internal variables fluctuate and as the two parameters change. The potential function can be written in terms of a variable(s) such as *Y(X)* for some performance dimension such as sales growth or rate of return. We assume therefore that what we are looking for is stable growth or return.

We have said that if a system is under the sway of an attractor, the system returns to stability after it is perturbed by an event, an employee absence, for example. Likewise, with the potential function, the system is assumed to return to the minimum of the potential function at the bottom of the well in Figure 4, the place in the diagram when the slope of the curve, the change in performance, the derivative *dY/dX*, is zero, if the system is perturbed. Thus, the bottom of the potential well in Figure 4 represents a stable attractor. It represents the realization of a system state that remains stable as it consistently appropriates resources such as profits, fees, taxes or human capital, for example.

To see how this relates to our parameter driven model of entrepreneurship, note that for a given *RRA* (i.e., when the parameter *b* is constant in Figure 3 above), the shape of the potential function varies as the *RAI* parameter, *a*, changes, the progression going from left to right shown in Figure 4. For each of the values of parameter *RAI* (*a* in the figure) in other words, depending upon the rate of accessible information, the shape of the function is different indicating differences in the attractor regimes at work in the company. Sometimes there is a single minimum; sometimes there are two minima. Being at the minimum, any small change - such as an employee calling out sick - might temporarily reduce performance, but the system would tend to quickly "roll back" to the point stability by having a team cover for the missing worker, again reaching the point of stability when the attractor is reached. The particular shape of the curve determines how much local fluctuation the system variable can endure while maintaining its stability.

| Insert Figure 4 about here |

Remember that the transformation of the potential "well" from left to right in the figure occurs as *RAI* increases, and that *RAI* increases as more and more information becomes accessible and is disseminated. This evolution of informational richness is represented by a "wave of change" associated with the perceived possibilities among the participants as this effect moves from left to right. Thus, as shown in panel 2, the social system can cross a threshold (entering the cusp region in Figure 2) where one attractor becomes two attractors, albeit of different performance potentials. The new one, represented in Panel 2 is depicted as an indentation in the well's wall. In other words, the new attractor is gaining ascendancy as distributed information about markets, technology, regulations, and social trends, becomes available to agents across the system. Although locally stable, its overall potential does not yet challenge current practice.

Panel 4 shows the first attractor still hanging on, reflecting the continuation of organizing in the old way of doing things even as new information would seem to imply that there is *RRA* that enables a

better way of organizing with greater realized performance. This just goes to show that the change in attractors is a serious challenge to the system's normal mode of being organized. Eventually, though, as *RAI* increases an entirely new operating model (as the system leaves the cusp region in Figure 2) takes over in Panel 5, an innovation not only in products but also in the business model itself. This new attractor will now dominate the system and determine individual behavior.

Beyond panel five and therefore outside the cusp of change, the old way is forgotten as the system moves forward in its new way of doing things. Importantly, however, the old way is not necessarily forgotten by the individuals involved. Experiences like these can be a source for valuable informational differences in individuals that can become the seeds of future innovations[4]. This leads to another two propositions:

Proposition 4: For given values of the parameters *RAA* and *RAI*, there is usually a single way to organize locally that is i) observable among the individuals within the system, and ii) achieves a stable performance potential for a given parametric situation. The exception occurs when the parameters indicate that the system is in the *cusp of change* in which case at least two alternative approaches to organizing can be observed, most often with different stable performance potential.

Furthermore, the relationships described in Proposition 4 can be formalized in a mathematical model as follows:

Proposition 5: Generative conditions that enable entrepreneurial innovation can be accurately represented by the cusp catastrophe model. The stable performance potential for entrepreneurial organizing in a given resource and information environment can be measured and assessed, and the relationship between performance, internal variables and the parameters *RAI* and *RRA* will be described using the quartic and cubic equations for the surfaces in a cusp catastrophe as follows:

---

[4] As we allude to in the Appendix, this same fact may limit the direct applicability of these mathematical techniques for human systems. New mathematical results may be needed to pursue these ideas more rigorously.

Performance potential Y associated with complex organizing is shown in equation [1] as a function of an internal variable X and parameters a, b, and k_i (normalization constants) as follows:

$$Y(X) = k_0 X^4 + k_1 b X^2 + k_2 a \qquad [1]$$

Where $a = RAI$, $b = RRA$ and the k_i are normalization constants. Stable organizing to realize performance potential is shown in equation [2] after coefficients are normalized:

$$dY/dX = 0 = X^3 + bX + a \text{ (derivative set to zero)} \qquad [2]$$

The reader should note that the equations in Proposition 5 are not arbitrary but are closely related to the Landau equations commonly used to describe phase transitions in physics (Haken, 2006). We are contributing to the field by applying this general result to innovation and entrepreneurship.

4. The role of emergence in entrepreneurial innovation at criticalization

Constraints (parameters) create the crucible wherein innovation occurs and attractors represent the organizing modes wherein that potential can be achieved (Hazy, 2009). What remains is the issue of how it is that people, resources, and technology can be organized in an ongoing adaptive fashion so as to approach the higher performing attractors particularly if they were originally caught up in the lower performing one, the path-dependency problem (Arthur, 2009). Schumpeter (1982: 65), for one pointed to *recombination* as the means by which innovation occurs: "As a rule, the new combinations must draw the necessary means of production from some old combinations … development consists primarily in employing existing resources in a different way, in doing new things with them." More recently, Arthur (2009) makes similar arguments for technology innovation. As Lichtenstein, et al. (2006) described, innovation in entrepreneurship involves the *emergence* - in the unique complexity science meaning of that term (see Appendix B) - of new modes of thinking and organizing.

Entrepreneurial ventures are examples of emergence to the extent that innovation in them can be described in terms of the arising of novel structures, novel practices, and novel properties; macro- or

collective-level dynamics; and a significant type of unpredictability. In nds models, the parameter

bifurcation scenario in the cusp of change indicates when emergence is likely to take place - as Cohen

and Stewart (1994: 234) have pointed out, "Mathematics wallows in emergent phenomena" and the

same can be said of social systems in general (see Sawyers, 2005). It also needs to be pointed out,

however, that emergence in different settings and at different phases of the same systems will involve

novelty generating mechanisms that may be significantly different from one another.

The host of recombinatory mechanisms found in emergence offer a much more powerful

source of novelty generation than the non-complexity based theories of technology innovation which

rely on gradual variations as in the Darwinian evolutionary model of innovation (for example, Van de

Ven and Garud, 1994) in which McKelvey (2004) has uncovered equilibrium assumptions. By

contrast, Arthur (2009), Arthur and Polack (2004), Fleming (2001), and Fleming and Sorenson (2001)

provide a more plausible account of the onset of technological innovation by using a complexity

science lens which stresses recombination operations acting on extant components. Fleming and

Sorenson quote Nelson and Winter (1982: 130) "the creation of any sort of novelty in art, science, or

practical life -- consists ... of a recombination of [extant] conceptual and physical materials."

Recombination has also been added to the tool-kit for chemists and biologists as a way to generate

innovative new compounds and sequences (see Beck-Singer and Weber, 2001). The emergence of

innovation can be seen as a search for better combinations of existing components.

It is important to point out that the understanding of emergence being used in our model is a

decided revision of earlier perspectives which associated it closely with the idea of self-organization,

the supposedly spontaneous arising of new systemic structures out of a system's own internal

dynamics (Nicolis, 1989; Nicolis and Prigogine, 1989). This older view of emergence, found in

organizational applications of the notion (e.g., see Chiles et al. 2004; MacIntosh and Maclean, 1999)

has served to highlight supposedly spontaneous re-organizing processes operating out of a system's

own resources. A closer inspection of actual research demonstrates that emergence is more appropriately seen as involving numerous constraining and other constructional operations including recombinatory ones (Anderson and Stein, 1987; Goldstein, 2002, 2006, 2007).

By consisting of recombinatory operations, emergence does not occur *ex nihilo* but at a level n recombining components on an already existing substrate level n-1 but within the parameters or constraints of level n+1 (see Salthe, 1993). Goldstein (2001, 2002, 2006) has generalized the recombinatory processes of emergence with the formalism of *self-transcending constructions* ("stc's") in which the suffix "self-", unlike its role in "self-organization", refers to how the lower level substrate order is transcended by novelty generating constructional operations. Stc's can be said to *complexify* systems by embedding information into the system through means of recombining routines and capabilities uncovered in shifts in strategic thinking of firms (Helfat, et al., 2007) to the point that outcomes become incommensurate to the substrate inputs. Recently this has been mathematically formalized by category theory in the work of Ehresmann and Vanderbresch (2007). The conceptual similarity of this idea to the emergence of innovations such as the microprocessor, the airplane, or even the hub-and-spoke transportation routing scheme is immediately clear.

The key role of recombinatory operations is related to *bricolage* as a potent source of innovation by entrepreneurs rather than merely relying on a grand plan (Baker, *et al.* 2003; Baker and Nelson, 2005; Garud and Karnoe, 2002). Recombining what is at hand in terms of streams of activity within and across different frames of use, production and governance, is clearly related to the parameter *RAI* since knowledge itself is recombined (Floyd and Wooldridge, 1999; Ghosh et al, 2009).

In fact, at the cusp of change, there are countless components at hand to be recombined in generating emergent innovations: organizational/community capabilities including leadership and followership expertise; operational proficiencies: knowledge management resources; organizational structures such as extant governance bodies; existing information technology and other technology

resources and know-how; capacities for knowledge and technology transfer; multiple funding streams; multiple organizational and personal missions; and the multifarious existing social networks in operation in collaborations, partnerships, community organizations, government bodies, businesses, universities and so forth. Far from being merely a spontaneous process, recombinatory innovation comes about through adroit facilitation of entrepreneurs in constructing a better way of doing things.

To be sure, many of these components may not be particularly noticeable at first, but complexity science has come very far in the detection and amplification of weak signals (see Goldstein et al., 2010) as well as the bifurcation explanatory model we propose here. This implies a proposition:

Proposition 6: The components of recombination activities will be more noticeable and more amenable for the emergence of innovation within a entrepreneurial venture if and only if the parametric position of the system places it within the cusp of change.

One way to operationalize this proposition is through a counting of components of novelty, perhaps, for example, using patents as found in Fleming (2001) and Fleming and Sorenson (2001).

4. 1 Constructing social networks for recombining routines and capabilities

One of the most important elements for recombination in the direction of social innovation is the use of social networks for recombining routines and capabilities. For instance, Yang (2004) proposes that one way to leverage social networks for entrepreneurial purposes is to focus on the *structural holes* in social networks (Burt, 1992), the locus of intersections of networks where nodes may not yet exist. This can have powerful resource availability and information benefits, can impact on *RRA* and *RAI* by bridging isolated islands of social capital, and can at the same time become the source of constraints on action that are brought about through social influence. As Ahuja (2000) has pointed out, firms must strike a balance between direct ties and indirect ties so as to reap the maximum benefits of their network, an insight related to Granovetter's (1983) research on weak-tie networks.

In relation to the recombining of social networks for innovation, entrepreneurs need to be

mindful of the paradox of embedded agency: actors acting in consort become catalysts for structural change within an institutional structure that in turn exerts pressures on these actors (DiMaggio, 1988, Garud*, et al.,* 2007; Holm 1995). Similarly, in their review of sixty-seven articles on institutional entrepreneurship, Leca, et al (2008) noted two areas for future research suggesting that research should explore how multiple actors with different resources coalesce and the conditions under which actors mobilize. Our model addresses both of these issues since *RRA* includes recombining social networks to take greater advantage of resources, which in turn generates more structure for coordination which then spurs on more cohesiveness and so on (Ramlogan & Consoli, 2008), and *RAI* similarly sets up conditions for better information flow by more effective linkage structures within network channels**.** Research has shown in a similar context that as inter-organizational networks form and evolve, formal ties based on equity yield over time to more informal ties based on trust and thus implying more open sharing of information (Gulati, 1995). This relates to findings that rather than maximizing the number of possible relationships, it may be better to seek "multiplex" relationships, characterized by a variety of exchanges and relationships where once again the effects on *RRA* and *RAI* are evident (Panzar et al., 2007; Provan & Sydow, 2008).

In researching the social network structure that expedited the rise of new businesses in Hungary after the fall of communism, Vedres and Stark (2009) identified a novel network facilitating the creation and successful implementation of innovation, a recombination of extant social networks into "intercohesion" networks which are named for the rich interaction between initially separate but internally cohesive social groups. Intercohesion occurs when the membership of these groups overlaps and interpenetrates in ways that capture the benefit of and the innovation potential available from Granovetter's (1983) mix of strong and weak ties. Such networks have a facility for recombining ideas and components from across sector boundaries. Vedres and Stark point to Lester's and Piore's (2004) investigation of new product innovations by combining disparate categories: cellular telephones as

hybrids of technologies from radio and telephony; medical devices blending biological science, engineering, and clinical need; and blue jeans which emerged at the interface of traditional work clothing and hospital/hotel laundry methods. These remarks imply a further proposition:

Proposition 7: A sudden system shift from a lower performing to a higher performing attractor within a cusp of change is positively related to the emergence of distinct and identifiable network structures, e.g., an intercohesion network.

5. An application of the model to entrepreneurial teams

One key for promoting entrepreneurship, we have argued, is the recognition of the crucial role of the parameters *RRA* and *RAI* in generating conditions for innovation by expediting the emergence of multiple attractors at criticalization so that new business models may emerge. For example, *RRA* might be influenced by a new discovery of a physical resource such as an oil field, or the invention of a new technology like deep water drilling, each of which presents new entrepreneurial opportunities. Similarly, *RAI* might be influenced by building a social network, developing strong associations with individuals with complementary skills, or using new communications technologies such a mobile applications. Let's assume as we did in Section 3 that the *RRA* has increased to the point where two distinct attractors may coexist, an indication that there are two distinct organizing approaches, the old way of doing things and a better one. The launch of the 3G and subsequent 4G wireless networks might present such an opportunity for example. For the transition to innovation to take hold, a couple of things must take place.

First, concerning *RAI*, informational differences must be recognized and brought to the surface. This would be equivalent to engendering an experiment-friendly atmosphere (Goldstein, et al, 2010) where seeds of novelty are expressed as informational differences are amplified and sustained even if they are not yet ready for prime time. Second, regarding *RRA*, synergies must be promoted among social networks to create more coordinated organizing options for the business. Taken together, these

behaviors are generative of a venture's future in the iterative manner described by Surie and Hazy

(2006) and shown in Figure 5. Members of an entrepreneurial team interact internally and externally

to gather and process information that is used to build iterative models of the opportunity and how to

organize to exploit it. Complexity researcher Guastello (2002) has identified cusp catastrophe patterns

in data associated with creativity and leadership in teams, an important prerequisite for innovation.

| Insert Figure 5 about here |

Another insight from our model involving entrepreneurial decision making is that just as there

are different possible attractors emerging during critical periods, there are also always at least two

pathways that such teams can take toward innovation. Up until now we have described the pathway

that leads to radical or punctuated change where an entrepreneur organizes and innovates to solve a

marketplace problem or address an opportunity. This is because we assumed a fixed level of *RRA*

wherein entrepreneurial teams "jump" (sometimes in a painful, iterative journey of discovery) to where

the venture moves forward as a whole toward a new model of organizing to exploit the opportunity.

With the *RAI* parameter at a high enough value, the team would be closely aligned and cognizant of all

available information so that all members are empowered to "jump" together.

However, it is also possible to build a venture by adjusting the *RRA* parameter and focusing on

one subset of available resources, e.g., breaking off a bite-size piece of the problem or opportunity in

order to promote change to a new attractor by incremental means. This is done by reframing the

perceived opportunity as a series as smaller problems that are more easily addressed - effectively

acting as if the value of *RRA* is lower - and thus reducing the perceived need for radical systemic

change. These smaller changes are then reassembled with the same recombination process described in

Section 4. Both of these pathways to change – the radical punctuation and the more continuous

incremental one can generate a change to a new attractor. This is depicted by the numbered pathways

in Figure 6 (an elaboration of Figure 3). In the figure, path #1 represents a punctuated change arising

from innovation whereas path #2 reflects bite-sized projects and a more continuous transformation.

| Insert Figure 6 about here |

Research has shown that entrepreneurial innovation can come about through either a leap of transformation, e.g., as shown in the textual analysis of entrepreneurial narratives studied by Lichtenstein et al. (2006), or through a more gradual, step-by-step approach as described by Arthur (2009) in the technology innovation case.

6. Conclusion: Lessons and Directions

A complexity science based model of entrepreneurial innovation offers several benefits. First, it acknowledges not only the actual complexity of real-world social, political, and economic systems within which entrepreneurial ventures unfold, it also explicitly considers such essential complex systems' features as social networks, interdependencies, unpredictable outcomes, emergence, and so forth. Further, it does so by offering a *systemic* perspective that integrates various enablers of success - RRA, RRA, performance potentials, recombinatory mechanisms, and iterated model building, all without appealing to supposedly charismatic founders. As such it is a platform for further investigation into the specific parameters driving entrepreneurship in many instantiations, within developed or developing economies, and for ventures with social as well as economic missions. Moreover, the model is grounded in the power of *radical* and nonlinear novelty-generating processes and practices that are required as research goes beyond equilibrium-based economic or evolutionary theories.

In addition, the model provides testable propositions and pertinent metrics to quantify different aspects of entrepreneurship. For example, a research agenda based upon the propositions in this paper might proceed as follows. To explore Propositions 1, and 2, researchers might use cross sectional surveys of individuals in both start-up businesses and those not participating to identify variables and scales that reflect the thresholds of the *RAA* and *RAI* parameters that lead to efforts at organizing new ventures in various industries. This work can be complemented by text analysis methods applied to the

narratives collected from the participants along the lines of the textual analysis of emergence that was described by Lichtenstein et al. (2006).

These scales could then be used as controls in field studies to explore those propositions aiming directly at the relationships among the parameters, the phenomenon of criticalization at bifurcations, and the cusp catastrophe as a useful model of innovation. Beyond these propositions, future work in this area can follow the lead of Ulrich (1995) who, in discussing the emergence of new inventions, has pointed to additional factors related to recombination mechanisms including integrality, coupling, and modularity. We can add here also the important challenge to "scale-up" venture enterprises since the notion of scaling is proving to be a major concern in complexity science research.

Through its study of criticalization phenomena, as well as research into the detection and amplification of weak signals and small differences, a complexity science model can also present insights into the often touted disruptive quality of innovation: "innovation is disruptive: it breaks established patterns of behavior" (Metcalfe, 1995: 1562). The challenge is to understand how this disruptive process works, and to generate guidelines on how to harness the recombination process we describe to provide the best possible outcomes.

Recombination that occurs between new technology and practical genius - as happened with the electric light bulb, microcomputer, Internet, and smart phone - have had a tremendous impact on society and the economic system. Likewise, when innovation is viewed through a wider lens, we believe that technology from complexity science when recombined with the practical genius of today's entrepreneurs will launch a new era of theory-driven research and entrepreneurial innovation.

Appendix A

As a prototypical example of a nonlinear dynamical system we describe the logistics map since it is also an important subject of study in its own right in the field of nds. In addition to having an important history in the study of deterministic chaos, it illustrates several important concepts. In

particular, the logistics map demonstrates how a phase space diagram can be made by plotting the state of a single variable for a system as the mapping is iterated over a series of time intervals; it shows the impact of a bifurcation inducing parameter, in this case the growth rate, k (see equation [A1]); and it demonstrates the emergence of bifurcated dynamics.

The mapping shown below is used as a simple representation of population dynamics year-to-year when the environment has a limited carrying capacity (of resources) to support the individuals in the population. In its simplest form, this equation is a "map" showing discrete changes in the variable X at discrete points in time where X at time $t+1$ depends upon X at time t and a single parameter k, and where the environment has a carrying capacity that is normalized to 1. The equation is written as:

$$X_{t+1} = k\, X_t\, (1 - X_t) \qquad\qquad\qquad\qquad\qquad\qquad \text{[A1]}$$

This 2nd degree mapping (it has a squared term) is sometimes displayed as an inverted parabolic shape (the "inverted U") that is normalized to be included in the unit interval. As the function iterates, it shows the rising and falling of population year-to-year for a given parametric setting. As the parameter k changes, indicating that the environmental context or constraints - in the prototypical case this is stated in terms of birth/death rates - are also changing, the resulting behavior of the system as it iterates changes often in striking ways. At low values of k, population stability is eventually achieved. At higher values, however, more complex results such as fluctuating population levels in a periodic oscillation ensue; at a high enough value for k, a type of deterministic unpredictability called deterministic chaos becomes evident (Hirsch et al, 2004).

The logistic map has been used with great success to depict the general characteristics of nonlinear dynamical systems mostly because, as a 2nd degree equation, it is the simplest case of parameterized nonlinearity (Feigenbaum, 1983; May, 1976) that can be used to approximate real world phenomena. We are certainly not claiming here that the rich dynamics of entrepreneurship can be captured in such a simple equation or with the comparatively simple phase space diagrams that the

logistics map alone implies. However, we do feel that the more inclusive results from nds that we use in this paper can provide useful insights which cannot otherwise be gained. For example, to address complex feedback loops at work in human interaction dynamics at points of bifurcation, we include the additional nonlinear dynamics at work within 3rd and 4th degree polynomials to identify two interacting parameters a and b. Certainly, these dynamics are entirely absent in traditional linear models.

To support the applicability of our approach even for complex human systems, we point to Takens (Sauer, 2005) who proved the remarkable result that much of the significant dynamics taking place among multiple variables in the phase space (as one would expect to see in a dynamical system that reflects human interaction) can be accurately reflected in a phase plot that is constituted when the values of a single variable are plotted over a series of time intervals. The trick, of course, is defining that single variable. At the same time, however, one must be careful when applying these purely mathematical results directly to human interaction dynamics. Takens' theorem depends upon some technical assumptions which may or may not apply when modeling human interaction dynamics. At the same time, these results do at least suggest that, to some extent, nonlinear models with one variable are useful for research agendas utilizing nds models to explore specific phenomena.

Appendix B

A major stream of research within complexity science is directly concerned with the novelty generating potency in nonlinear complex systems, namely the study of *emergence* consisting of varied recombinatory and related operations leading to unexpected properties and structures (see, Crutchfield, 1993, 1994; Goldstein, 1999, 2006, 2007). Emergence has been mathematically formalized in Holland's (1998) treatment of the computational emergence that characterizes artificial life (see Adami, 1998). Holland's approach is based upon recursive operations that he has termed Constrained Generating Procedures (CGP). The recursive operations he proposes act on a set of input combinations of components of the system to generate new components which themselves become subject to the

same recursive operations in the next iteration. In equation form it is the mapping:

$$f_c: I_c \times S_c \longrightarrow S_c \qquad \text{[B1]}$$

Where I_c = possible input combinations at time t, S_c = states of system at time t, and f_c is defined in

[B1] so that:

$$f_c(t+1) = f_c(I_c(t), S_c(t)) \qquad \text{[B2]}$$

Holland complements this recombinatory procedure for emergence with other novelty generating

operators such as his genetic-algorithms which recursively "mutate" at each generation of recursion. A

related formalism of emergent recombination is offered in a model based on mathematical logic in

Goldstein (2002). Further, we posit that locally f_c in [B2] is of the form described in Proposition 5,

since changing form is implied and can be modeled at multiple levels of scale.

References

Abraham, R. 1982. Dynamics: The Geometry of Behavior (Four Volumes). Aerial Press, Santa Cruz, CA.

Adami, C. 1998. An Introduction to Artificial Life. Springer Telos, New York.

Ahuja, G. 2000. Collaboration networks, structural holes, and innovation: A longitudinal study, Administrative Science Quarterly, 45, 425-455.

Allen, P., Maguire, S., McKelvey, B. 2011. The Sage Handbook of Complexity and Management. Sage Publishing, London, UK.

Anderson, P., Stein, D. 1987. Broken symmetry, emergent properties, dissipative structures, life: Are they related? in: Yates, F. (Ed), Self-organizing systems: The Emergence of order. Plenum Press, New York, pp. 445--457.

Arnold, V. I. 1991. Catastrophe Theory, third ed. Springer-Verlag, Berlin.

Arthur, W. B. 2009. The Nature of Technology. Free Press, New York.

Arthur, W. B., Polak, W. 2004 The Evolution of Technology Within a Simple Computer Model, Santa Fe Institute Working Paper 2004-12-042.

Ashby, W. R. 1956. An Introduction to Cybernetics. Wiley, New York.

Baker, T., Miner, A. S., and Eesley, D. T. 2003. Improvising firms: Bricolage, account giving and improvisational competencies in the founding process. Research Policy, 32, 255-276.

Baker, T., Nelson, R. 2005 Creating something from nothing: Resource construction through entrepreneurial bricolage. Administrative Science Quarterly, 50, 329-366.

Bateson, G. 2000. Steps to an Ecology of Mind: Collected Essays in Anthropology, Psychiatry, Evolution, and Epistemology. University of Chicago Press, Chicago.

Beck-Sickinger, A., Weber, P. 2001. Combinatorial Strategies in Biology and Chemistry. Wiley, Chichester, England.

Berge, P., Pomeau, I., Vidal, C. 1984. Order within Chaos: Towards a Deterministic Approach to Turbulence. NY: John Wiley and Sons. New York.

Burt, R. S. 1992. Structural Holes Harvard University Press, Cambridge, MA.

Camazine, S., et al. 2001. Self-organization in Biological Systems. Princeton University Press, Princeton, NJ.

Chiles, T., Meyer, A., Hench, T. 2004 Organizational emergence: The origin and transformation of Branson, Missouri's musical theaters. Organization Science, 15, 499-519.

Cohen, J., Stewart, I. 2000. The Collapse of Chaos: Discovering Simplicity in a Complex World. Penguin Books, New York.

Crutchfield, J. 1993. The Calculi of Emergence: Computation, Dynamics, and Induction. Santa Fe Institute Working Paper # 94-03-016.

Crutchfield, J. 1994 Is Anything Ever New? Considering Emergence. Santa Fe Institute Working Paper #94-03-011.

Dacin, P. A., Dacin, M. T., and Matear, M., 2010. Social Entrepreneurship: Why we don't need a new theory and how we move forward from here. Academy of Management Perspectives, 24(3), 37-57.

Devarajan, S. and Delfin, S. 1998. The simplest dynamic general-equilibrium model of an open economy. Journal of Policy Modeling, 20, 677-714.

DiMaggio, P. J. 1988. Interest and agency in institutional theory, in: L. Zucker (Ed.), Institutional Patterns and Organizations. Ballinger,Cambridge, MA. pp. 3-22.

Dodds, P. S., Watts, D. J. 2005. A generalized model of social and biological contagion. Journal of Theoretical Biology, 232, 587-604.

Eckhardt, J. T., Shane, S. A. 2003. Opportunities and entrepreneurship. *Journal of Management,* 29, 333-349.

Ehresmann, A. C., Vanbremeersch, J. P. 2007. Memory Evolutive Systems: Hierarchy, Emergence, Cognition. Elsevier, NY.

Epstein, J. M. 1997. Nonlinear Dynamics, Mathematical Biology, and Social Science. Addison-Wesley, Reading MA.

Feigenbaum, M. 1983. Universal behavior in nonlinear systems, in: Campbell, D., Rose, H. (Eds.). Order in chaos. North-Holland, Amsterdam (Reprinted from Physica D, 7, 16-39.

Fleming, L. 2001, Recombinant uncertainty in technological search. Management Science, 47, 117-132.

Fleming, L., Sorenson, O. 2001. Technology as a complex adaptive system: Evidence from patent data. Research Policy, 30, 1019-1049.

Floyd, S. W., Wooldridge, B. 1999. Knowledge creation and social networks in corporate entrepreneurship: The renewal of organizational capability. Entrepreneurship Theory and Practice, 123-143.

Garud, R., Hardy, C., Maguire, S. 2007. Institutional entrepreneurship as embedded agency: An introduction to the special issue, Organization Studies, 28, 957-969.

Garud, R., Karnoe, P. 2002. Complexity, bricolage, and technology, in: Lissack, M. (Ed.), The Interaction of Complexity and Management. Quorum Books, Westport, CT.

Gell-Mann, M. 2002. What is complexity? in" Curzio, A., Fortis, M. (Eds.), Complexity and Industrial Clusters: Dynamics and Models in Theory and Practice. Physica-Verlag, Berlin, pp. 13-24.

Ghosh, A., et al. 2009. Recombination experience: A study of organizational learning and its innovation impact. Paper presented at the 29th Annual Conference of the Strategic Management Society, Washington, D.C., February, 9, 2009. Available at http://www-management.wharton.upenn.edu/pennings/documents/SMSSubmission2009-v41.pdf accessed 18 October 2009).

Goldstein, J. 1988. A far-from-equilibrium systems approach to resistance to change. Organizational Dynamics, 17, 16-26.

Goldstein, J. 1990. A nonequilibrium, nonlinear approach to organizational change. System Dynamics '90, vol. 1, 425-439.

Goldstein, J. 1999. Emergence as a construct: History and issues. Emergence: Complexity Issues in Organization And Management, 1, 49-62.

Goldstein, J. 2001. Emergence, radical novelty, and the philosophy of mathematics. in: Sulis, W., Trofimova, I. (Eds.), Nonlinear Dynamics in the Life and Social Sciences, IOS Press, vol. 320 - NATO Science Series, Amsterdam, pp. 133-152.

Goldstein, J. 2002. The singular nature of emergent levels: Suggestions for a theory of emergence, Nonlinear Dynamics, Psychology, and Life Sciences, 6, 293-309.

Goldstein, J. 2006. Emergence, creative process, and self-transcending constructions, in: Richardson, K. (Ed.), Managing Organizational Complexity: Philosophy, Theory,   and Application. Information Age Press, Greenwich, CT, pp. 63-78.

Goldstein, J. 2007. A new model for emergence and its leadership implications,   in: Goldstein, J. Hazy, J.K., Lichtenstein, B. (Eds.), Complex Systems Leadership Theory. ISCE Publishing, Mansfield, MA, pp. 61-92.

Goldstein, J. Hazy, J. K., Lichtenstein, B. 2010. Complexity and the Nexus of Leadership: Leveraging Nonlinear Science to Create Ecologies of Innovation. Palgrave-Macmillan, Englewood Cliffs, NJ.

Goldstein, J. A., Hazy, J. K., Silberstang, J. 2010. A Complexity model of social innovation in social enterprise. Journal of Social Entrepreneurship, 1, 101-125.

Granovetter, M. 1983. The strength of weak ties: A network theory revisited. Sociological Theory, 1, 201-233.

Gulati, R. 1995. Does familiarity breed trust? The implications of repeated ties for

    contractual choice in alliances. Academy of Management Journal, 38, 85-112.

Guastello, S. J. 1992. Clash of the paradigms: A critique of an examination of the polynomial

    regression technique for evaluating catastrophe theory hypotheses. Psychological Bulletin,

    111, 375-379.

Guastello, S. J. 2002. Managing Emergent Phenomena: Nonlinear Dynamics in Work Organizations.

    Lawrence Erlbaum, Mahwah, NJ.

Haken, H. 2006. Information and Self-organization: A Macroscopic Approach to Complex Systems,

    third ed. Springer, Berlin.

Hazy, J. K. 2008. Toward a theory of leadership in complex systems: Computational modeling

    explorations. Nonlinear Dynamics, Psychology, & Life Sciences, 12, 281-310.

Hazy, J. K. 2009. Innovation Reordering: Five Principles for Leading Continuous Renewal, in:

    Schlomer, S.,Tomaschek, N (Eds.), Leading in Complexity: New Ways of Management

    Systemische Forschung im Carl-Auer Verlag, Heidelberg, Germany, pp. 40-56.

Helfat, C. E., et al. 2007. *Dynamic Capabilities: Understanding Strategic Change in Organizations.*

    Blackwell, Malden, MA.

Henderson, J.M., Quandt, R.E. 1980. Microeconomic Theory: A Mathematical Approach. McGraw-

    Hill Bookm New York.

Hirsch, M. W., Smale, S., Devaney, R. L., 2004. Differential Equations, Dynamical Systems and an

    Introduction to Chaos third edition, vol. 60. Elsevier, Amsterdam.

Holland, J., 1998. Emergence: From Chaos to Order. Addison-Wesley, Reading, MA.

Holm, P., 1995. The dynamics of institutionalization: Transformation processes in Norwegian

    fisheries. Administrative Science Quarterly, 40, 398-422.

Jost, J. 2005. Dynamical Systems: Examples of Complex Behaviour. Springer, New York.

Kirzner, I. M., 1985. Discovery and the Capitalist Process. University of Chicago Press, Chicago.

Krugman, P. 1996. The Self-organizing Economy (Mitsui Lectures in Economics). Wiley-Blackwell, New York.

Laughlin, R. 2006. *A Different Universe: Reinventing Physics from the Bottom Down.* Basic Books, New York.

Leca, B., Battilana, J., Boxenbaum, E., 2008. Agency and institutions: A review of institutional entrepreneurship. Working Paper 08-096, Harvard Business School. Available at http://hbswk.hbs.edu/item/5943.html (accessed 5 November 2009).

Lee, J., Venkataraman, S., 2006. Aspiration, market offerings, and the pursuit of entrepreneurial opportunities. Journal of Business Venturing, 21, 107-123.

Lester, R. K., Piore, M. J., 2004. Innovation: The Missing Dimension. Harvard University Press, Cambridge, MA.

Levesque, M., 2004. Mathematics, theory and entrepreneurship. Journal of Business Venturing, 19(5), 743-765.

Levinthal, D., 1997. Adaptation on Rugged Landscapes. Management Science, 43(7), 934-950.

Lichtenstein, B., Carter, N., Dooley, K., and Gartner, W. (2007), "Dynamics of organizational emergence: Pace, punctuation, and timing in nascent entrepreneurship," *J. of Business Venturing*, 22(2): 236-261.

Lichtenstein, B., Dooley, K., Lumpkin, G., 2006. Measuring emergence in the dynamics of new venture creation. Journal of Business Venturing, 21, 153– 175.

MacIntosh, R., MacLean, D., 1999. Conditioned emergence: A dissipative structures approach to transformation. Strategic Management Journal, 20, 297-316.

Mackay, D. 1969, Information, Mechanism, and Meaning. MIT Press, Cambridge, MA.

May, R. 1976. Simple mathematical models with very complicated dynamics. Nature, 261, 459-467.

McKelvey, B., 2004. Toward a complexity science of entrepreneurship, Journal of Business Venturing, 19, 313-341.

Metcalfe, J.S., 1995. The design of order: notes on evolutionary principles and the dynamics of innovation. Revue e´conomique, 46, 1561–1583.

Morlacchi, P. 2007. Systems of innovation. Presentation at the Skoll World Forum, 29 March.

Nelson, R., Winter, S., 1982. An Evolutionary Theory of Economic Change. Belknap Press. Cambridge, MA.

Nicolis, G., 1989. Physics of far-from-equilibrium systems, in: P. Davies (Ed.), The New Physics. Cambridge University Press. Cambridge, U.K., 316-347.

Nicolis, G., Prigogine, I. 1989. Exploring Complexity: An Introduction. W. H. Freeman and Company, New York.

Page, S., 2007. The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies. Princeton University Press, Princeton, NJ.

Panzar, C., Hazy, J.K., McKelvey, B., Schwandt, D. R., 2007. The paradox of complex organizations: Leadership as integrative influence. In J. K. Hazy, J. Goldstein & B. B. Lichtenstein (eds.). Complex Systems Leadership Theory, ISCE Publishing, Mansfield, MA, pp. 345-370.

Provan, K. C. and Sydow, J., 2008. Evaluating inter-organizational relationships, in: Cropper, S., et al (Eds.), The Oxford Handbook of Inter-organizational Relations, Oxford University Press, Oxford, U.K., 691-718.

Ramlogan, R., Consoli, D., 2008. Knowledge, understanding and the dynamics of medical innovation. MPRA Paper No. 9112. Manchester, UK: Manchester Institute of Innovation Research. Available at http://mpra.ub.uni-muenchen.de/9112/ (accessed 5 November 2009).

Rumler, F. 1999. Computable General Equilibrium Modeling: Numerical Simulations In a 2-Country

Monetary General Equilibrium Model. Working Paper No. 65, Vienna University of

Economics and Business Administration. Available at

http://www.wu.ac.at/inst/vw1/papers/wu-wp65.pdf (accessed 2 October 2010).

Salthe, S., 1983. Development and Evolution: Complexity and Change in Biology. MIT Press,

Cambridge, MA.

Sarason, Y., Dean T., Dilliard, J. F. 2006. Entrepreneurship as the nexus of individual and

opportunity: A structuration view. Journal of Business Venturing, 21, 286-305.

Sauer, T. 2005. Embedding methods, in: Scott, A. (Ed.), Encyclopedia of Nonlinear Science. Taylor

and Francis, New York, 260-261.

Sawyers, K., 2005. Social Emergence: Societies as Complex Systems. Cambridge   University

Press, New York.

Schumpeter, J. A., 1934, 1982. The Theory of Economic Development. Transaction Publishers, New

Brunswick, NJ.

Scott, A. (Ed.) 2005. Encyclopedia of Nonlinear Science. Taylor and Francis, New York.

Siggelkow, N., Rivkin, J., 2005. Speed and search: Designing organizations for turbulence and

complexity, *Organization Science*, 16, 101-122.

Smith, L., 2002, Economies and markets as complex systems, *Business Economics,* 37(1), 46-53.

Surie, G., Hazy, J. K., 2006. Generative leadership: Innovation in complex environments. Emergence:

Complexity & Organization, 8(4), 13-26.

Thom, R., 1989. Structural Stability and Morphogenesis: An Outline of a General Theory of Models.

Addison-Wesley, Reading, MA.

Tufillaro, N. B., Abbot, A., Reilly, J. 1992. An Experimental Approach to Nonlinear Dynamics and

Chaos. Perseus, New York.

Uhl-Bien, M., Marion, R., and McKelvey, B., 2007. Complexity Leadership Theory: Shifting

leadership from the industrial age to the knowledge era. The Leadership Quarterly, 18(4), 298-318

Ulrich, K., 1995. The role of product architecture in the manufacturing firm. Research policy, 24, 419–440.

Vaghely, I. P., Julien, A., 2010. Are opportunities recognized or constructed?: An information perspective on entrepreneurial opportunity identification. Journal of Business Venturing, 25, 73-86.

Van de Ven, A. H, Garud, R., 1994. The coevolution of technical and institutional events in the development of an innovation, in: Baum, J., Singh, J. V., (Eds.), *Evolutionary Dynamics of Organizations*, Oxford University Press, Oxford, UK, pp. 425-450.

Vedres, B., Stark, D., 2009. Opening Closure: Intercohesion and Entrepreneurial Dynamics in Business Groups. MPIfG Discussion Paper 2009 (3) Max-Planck-Institut für Gesellschaftsforschung. Available at http://www.personal.ceu.hu/staff /Balazs_Vedres/research/PartB1_INTERCOHESION.pdf Accessed 7/10/09.

Yang, H., 2004. Leveraging social networks to cultivate entrepreneurial orientation: An organizational embeddedness perspective. Presented at the 2004 Midwest Academy of Management Conference, Minneapolis, MN. Available at http://cobacourses.creighton.edu/MAM/2004/papers/yang.doc (accessed 11 November 2009).

Weiss, N. O., 1987. Dynamics of Convection. in: Berry, M. et al. Dynamical Chaos. Princeton University Press, Princeton, NJ.

Williamson, O.E., 1975. Markets and Hierarchies: Analysis and Antitrust Implications. The Free Press, New York.

Zeeman, E. C., 1977. Catastrophe Theory: Selected Papers 1972-1977. Addison-Wesley, Reading,

MA: Publishing Company, I